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PRIME CORDIAL LABELLING OF BOW GRAPHS

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ABSTRACT:

Graph theory as a formal field of mathematics was **invented by Leonhard Euler**, a Swiss mathematician, in the 18th century.
Euler's groundbreaking work on graph theory is most famously associated with the **Seven Bridges of Königsberg** problem, which he solved in 1736.Shell graph is a shell **layered structure** in graph theory where nodes are grouped into concentric shells based on their distance from a central point.
Prime Cordial Labeling is a concept in **graph theory** that deals with the labeling of the vertices and edges of a graph in a particular way to satisfy certain conditions related to prime numbers.It involves labeling the vertices and edges of a graph with non-negative integers, such that the differences between the labels of adjacent vertices are prime numbers or zero. The graph must satisfy a specific condition for it to be considered a **prime cordial graph**.
Prime cordial labeling was introduced by **V. K. Jain** and **M. K. Chaurasia** in 2005. They introduced this concept as a part of their research in graph labeling theory, which explores how to assign labels (usually integers) to the vertices and edges of a graph under certain constraints
The term **"bow graph"** is not a widely recognized or standardized concept in the field of graph theory. However, based on the types of structures that might be referred to as "bow graphs" (such as bowtie graphs or bow-shaped graphs), the **bowtie graph** has a more established history in the study of networks and graphs..The formal introduction of **bowtie structures** in the study of networks is generally attributed to research in the **late 20th century**, especially in the context of understanding **network topologies** and **internet structures In this paper we have proved bow graphs admits prime cordial labelling.**

INTRODUCTION:

prime cordial labeling is a theoretical concept primarily studied within graph theory, its applications span a variety of fields where graphs and networks play a critical role. These includes **Communication networks** for routing and data flow optimization, **Cryptography** for enhancing security protocols, **Social network analysis** for understanding influence and community structures, **Biological networks** to model interactions and dependencies., **Optimization problems** in scheduling, resource distribution, and graph coloring.
Bow graphs are used in a wide range of fields, especially when analyzing complex networks with a central component that is highly interconnected, surrounded by peripheral components with weaker connections. These graphs are useful in areas such as network theory, social sciences, biology, transportation, logistics, and cybersecurity, where understanding the structure of connections can provide insights into how systems function, evolve, or fail.
In this paper we have proved Bow graphs admits prime cordial labelling.
Key words: Bow graph, prime cordial, labelling, vertex, edges, core

DEFINITION:

prime cordial labeling is a type of labeling where:

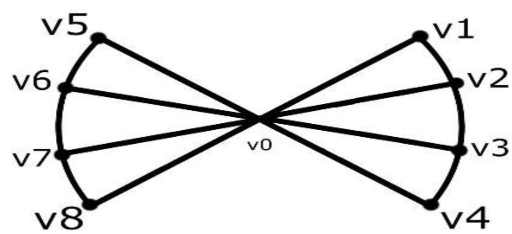
- The graph has a "cordial" labeling (odd/even balance),
- The edge labels (differences between adjacent vertex labels) are prime numbers.

This concept is often studied in the context of number theory and graph theory, exploring how the structure of a graph can be labeled in a way that satisfies both the cordial and prime number conditions.

Definition :

BOW-tie STRUCTURE:

- A **bow-tie structure** refers to a type of graph that resembles the shape of a bow-tie, with the main component being a strongly connected part (like the "knot" of the bow), and two other parts representing the "wings." This could describe a network where most nodes are connected to a central strongly connected component.
- This is used to model structures where certain nodes or regions are tightly interconnected while other regions might have weaker or more limited connectivity



:Fig.1: The Bow Graph

The centre strongly connected component represented by c1,c2,c3,..............cm and every node in the core can reach every other node within the core.

There are edges connecting the nodes in the core, making it strongly connected

MAIN RESULT:

In this section, we have proved that the Bow graph admits Prime Cordial labelling.

Theorem

Bow graphs admits Prime Cordial labelling.

Proof :

A bow structure in graph theory could also describe any graph where some vertices and edges create a "bow" shape, possibly with curves or other geometric shapes.

Let V0,V1,V2,................Vn be the vertices of bow graph and vo be the apex of the bow graph. Let p denotes the number of vertices and q denotes the number of edges.

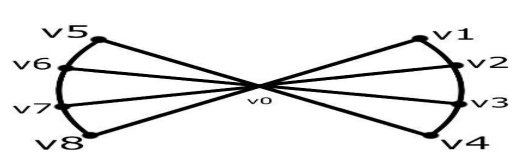


Fig .2: The bow graph with 8 vertices

Illustration : we illustrate the proof of the above theorem as follows:
Prime Cordial labelling applied in bow graph with 24 vertices.

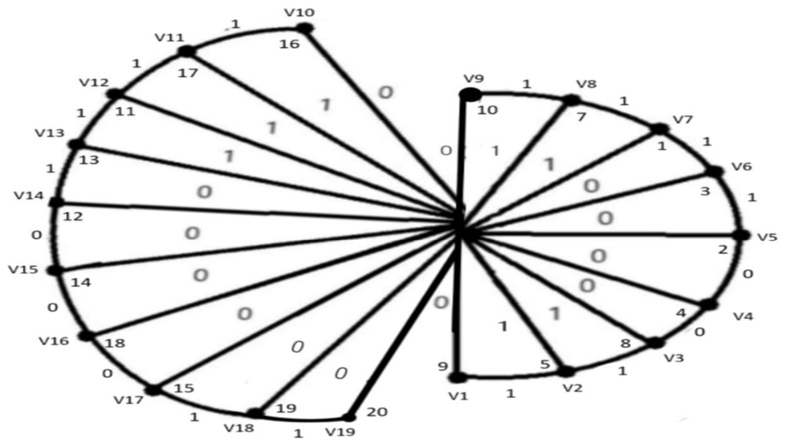


Fig.3: Prime Cordial labelling of Bow graphs.

Edge labelling of Prime Cordial labelling of bow graphs.

1.$v_0 - v_1 = 0$

2.. $v_{p-2} - v_{p-1} = 0$

3. $v_0 - v_i = 0$ i= 3 to 6

4, $v_0 - v_j = 0$ j= p-3 to p-1

5. $v_{i+2} - v_{i+3} = 0 \; i = 1,2$

6. $v_{p-4} - v_{p-3} = 1$

7. $v_i - v_{i+1} = 1, i = 1,2$

8. $v_i - v_{i+1} = 1, i = 5,6,7,8,9$

9. $v_{p-4} - v_{p-7} = 1.$

From the above equations 1 to 10 we have proved the prime cordial labelling of bow graphs.

CONCLUSION:

In this paper we have illustrated prime cordial labelling of bow graphs. We would be interesting to prove other prime cordial labelling in shell related family of graphs and also we would like to prove other labelling in shell related graphs.

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